MIDTERM: B3 REPRESENTATION THEORY

Date: 23$^{\text{rd}}$ **Sept 2022**

All representations considered here are of finite groups and over complex numbers. Total points is **110** and the maximum you can score is **100** points..

(1) (10+10=20 points) Let $\rho : G \to GL(V)$ be a representation. Show that the endomorphism $f = \sum_{g \in G} |g|\rho(g)$ of V is G-equivariant. Here $|g|$ denotes the order of g in G. If V is irreducible then compute f.

(2) (15+5=20 points) Let G be a finite group and V be the regular representation of G. Show that $\mathrm{Hom}(V, V)$ is isomorphic to $V \otimes V$. Show that this doesnot hold for every G-representation V.

(3) (10+10=20 points) Let D_4 be the dihedral group of order 8. How may irreducible representations D_4 has? Write down their dimension. Compute the number of four dimensional representations $D_4 \times (\mathbb{Z}/10\mathbb{Z})$.

(4) (5+15=20 points) Let G be a finite group acting on a set X. Define the associated permutation representation of G. Let V be the permutation representation of S_3 associated with the action of S_3 on $\{1, 2, 3\}$. Decompose $Sym^2 V$ and $Ext^2 V$ as direct sum of irreducible representations.

(5) (5+10+15=30 points) Define induced representation. Let G be a group and H be a subgroup of G. Let W be the trivial representation of H and $V = \mathrm{Ind}_H^G W$. Let $K = \cap_{g \in G} g^{-1}Hg$. Show that every element of K acts trivially on V. If G/K is abelian show that V is a direct sum of one dimensional subrepresentation.